Appendix B. Background Bias Estimation by χ^2-square Test.

Given the χ^2-square test and Eq. 23 with the slope parameter $a = 1$, we have,

$$\chi^2 = \sum_{k=1}^{N} \frac{\left(G_k - (R_k + b)\right)^2}{\sigma_{R_k}^2 + \sigma_{G_k}^2} = \sum_{k=1}^{N} \frac{\left(G_k - (R_k + b)\right)^2}{\sigma_{SR_k}^2 + \sigma_{SGk}^2 + \sigma_{BR}^2 + \sigma_{BG}^2}$$

$$= \sum_{k=1}^{N} \frac{\left(G_k - (R_k + b)\right)^2}{c^2 \mu_{R_k}^2 + c^2 \mu_{Gk}^2 + \sigma_{BR}^2 + \sigma_{BG}^2} = \sum_{k=1}^{N} \frac{\left(G_k - (R_k + b)\right)^2}{2c^2 \mu_{R_k}^2 + \sigma_{BR}^2 + \sigma_{BG}^2} \tag{B1}$$

Replacing the mean in the denominator by its null-hypothesis estimator $(R_k + G_k)/2$, and the red and green background variances by their sample variances $\hat{\sigma}_{BR}^2$ and $\hat{\sigma}_{BG}^2$, yields

$$\chi^2 = \sum_{k=1}^{N} \frac{2\left(G_k - (R_k + b)\right)^2}{c^2 (R_k + G_k) + 2\hat{\sigma}_{BR}^2 + 2\hat{\sigma}_{BG}^2} = \sum_{k=1}^{N} \frac{2\left(G_k - (R_k + b)\right)^2}{w_k} \tag{B2}$$